Effective Classroom Teaching of Optimal Control and Optimization using MATLAB

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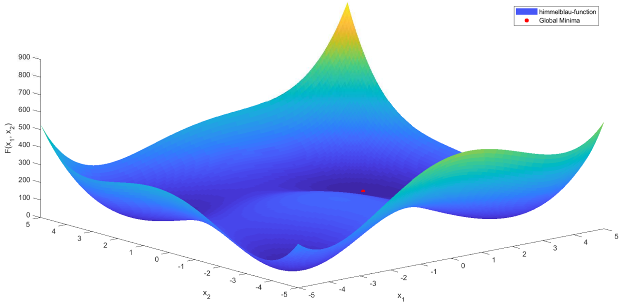
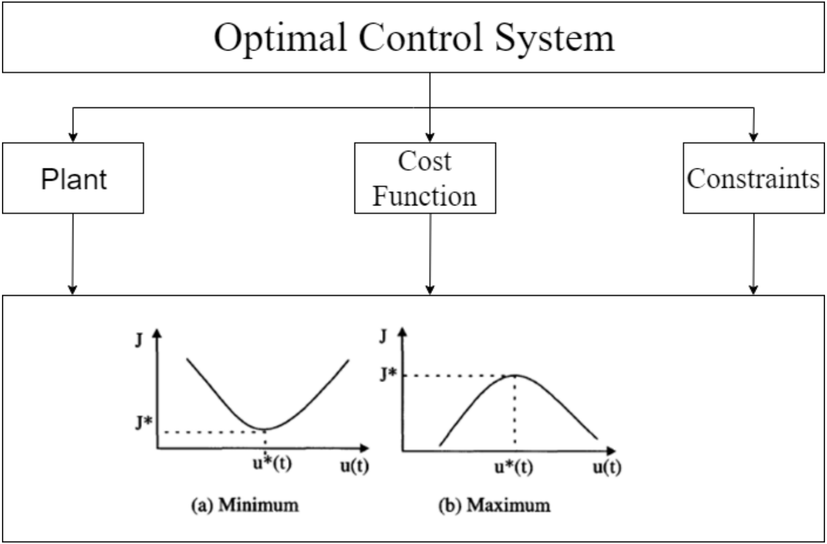
Optimal Control

- Finding values of the variables that optimize (minimize or maximize) the objective function while satisfying the constraints



Ingredients of Optimal Control Problem

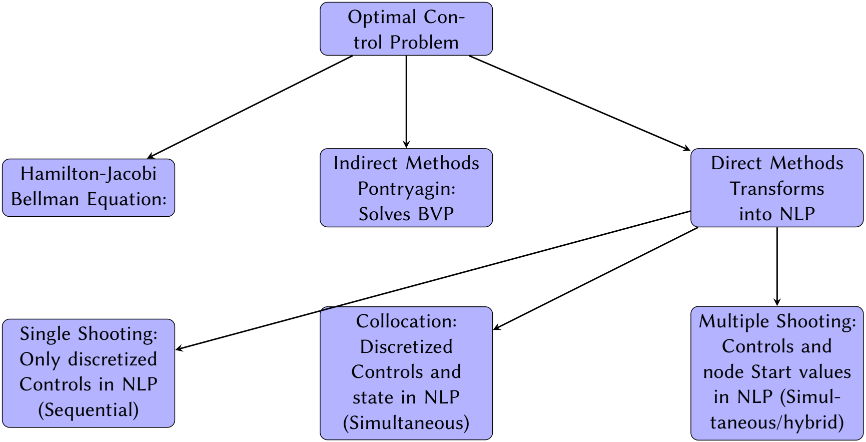


Methods to Solve OCP

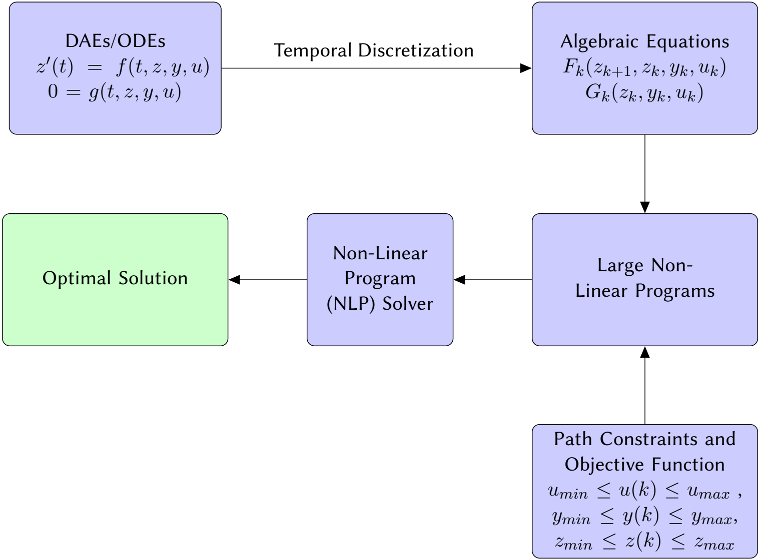
Analytical Methods – Pontryagins maximum principle

Numerical Methods

Numerical Methods to Solve OCP



Optimal Control Problem Flow Chart



Example Problem: Time Optimal Rocket Problem (Time Optimization Problem)

$$\min_{U} \quad t_f$$

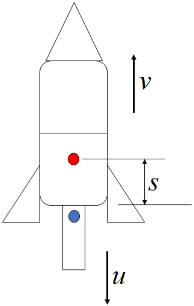
Subject to

$$\dot{s(t)} = v(t)\ ; \dot{v(t)} = \frac{(u(t) - 0.02 * v(t)^2)}{m(t)}$$

$$\dot{m(t)} = -0.01 * u(t)^2\ ;\ t \in \begin{bmatrix} 0 & t \end{bmatrix}$$

$$s(0) = 0;\ v(0) = 0;\ m(0) = 1;$$

$$s(t_f) = 10;\ v(t_f) = 0$$

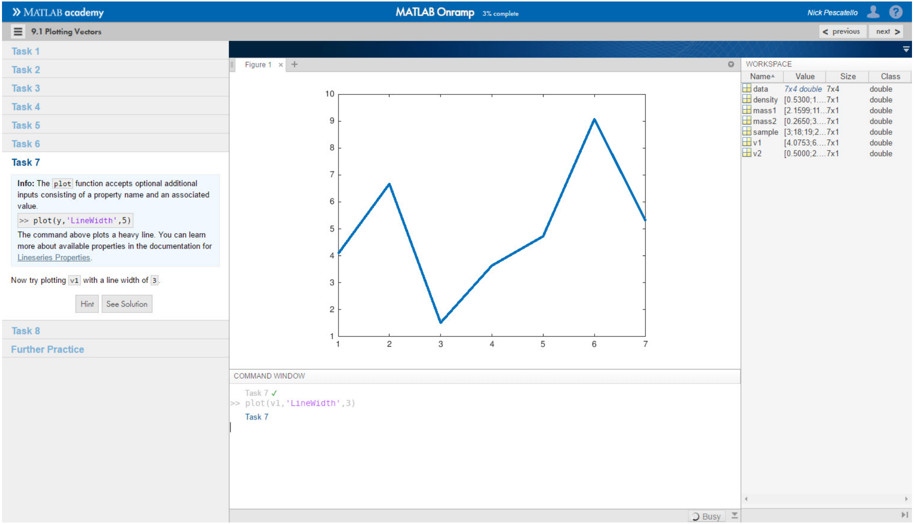


Bounds

$$-0.1 \leq v \leq 1.7;\ -1.1 \leq u \leq 1.1;\ 5 \leq T \leq 15$$

Approach: MATLAB and Simulink Onramp

To provide a brief introduction to the MATLAB language and to give students hands-on MATLAB experience via the use of an integrated, web-based version of MATLAB, as shown below.



Approach: MATLAB and Simulink Onramp Report

Progress Report

Name:	Sayli Patil
Course:	MATLAB Onramp
Progress:	100% complete (as of 17-Dec-2018)

Chapters

1. Course Overview 100%
2. Commands 100%
3. Vectors and Matrices 100%
4. Importing Data 100%
5. Indexing into and Modifying Arrays 100%
6. Array Calculations 100%
7. Calling Functions 100%
8. Obtaining Help 100%
9. Plotting Data 100%
10. Review Problems 100%
11. MATLAB Scripts 100%
12. Logical Arrays 100%
13. Programming 100%
14. Final Project 100%
15. Survey 100%

Classroom teaching approach: MATLAB Live Script: Interactive approach

Using Conjugate Gradient Method in MATLAB script.

We will try to find optimal solution of function

$f = x(1) - x(2) + 2 * x(1)^2 + 2 * x(1) * x(2) + x(2)^2;$

using Conjugate Gradient Method.

Initialize the values

```
clear all
clc
imax=0;
iter=1000;
x=[0 0] % Initial Value
xi = x' % Transpose
iteration=1;
```

Finding the Gradient of the function for the initial value:

```
gradf=grad_cost(@costfunc,xi) % Using grad_cost function
                               %  which is user defined.
```

Steepest direction

```
Si=-gradf
```

Get the function in the form of Lambda

```
syms lamda;
X=xi+lamda*Si;
```

```
x = 1×2
        0     0

xi = 2×1
        0
        0

gradf = 2×1
        1
       -1

Si = 2×1
       -1
        1

lamda = 1

x_new_i =
```

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

Teaching Optimal Control using Live Script

Check the condition if the solution is optimal

```
x_new_i=xi+ lamda*Si
delf_i= grad_cost(@costfunc,x_new_i);
if delf_i==0
    fprintf('Solution is optimum');
    fprintf('\n');
    fprintf('Total Number of Iterations = %d\n', imax );
    return;
end
```

Running the loop for finding the Optimal Solution

```
for i=1:iter
gradf=grad_cost(@costfunc,x_new_i);
gradf_old=grad_cost(@costfunc,xi);
Si=(-gradf)+Si*((gradf')*(gradf)*(inv((gradf_old')*(gradf_old))));
syms lamda;
L=x_new_i+lamda*Si;
f=costfunc(L);
K=diff(f);
lamda=solve(K,lamda);
x_new=x_new_i+ lamda*Si;
delf_i= grad_cost(@costfunc,x_new);
xi=x_new_i;
if delf_i==0
    fprintf('Solution is optimum');
break;
end
fval_old=costfunc(x_new_i);
fval_new=costfunc(x_new);
```

```
2   -1

Si = 2×1
    -1
     1

lamda = 1

x_new_i =
```

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

```
Solution is optimum
```

Teaching Optimal Control using Live Script

Check for stoping criterion

```
tol_1=abs((fval_new-fval_old)/fval_old);
if(tol_1<1e-6)
break;
end
x_new_i=x_new;
end
```

Printing the final values and number of iterations

```
imax=i+iteration ;
fprintf('\n');
fprintf('Solution by ConjugateGradient Method \n');
fprintf('\n');
fprintf('Total Number of Iterations = %d\n', imax )
lamda
x_new
fval=costfunc(x_new)
```

```
Solution is optimum


Solution by ConjugateGradient Method

Total Number of Iterations = 2
lamda =
```

$$\frac{1}{4}$$

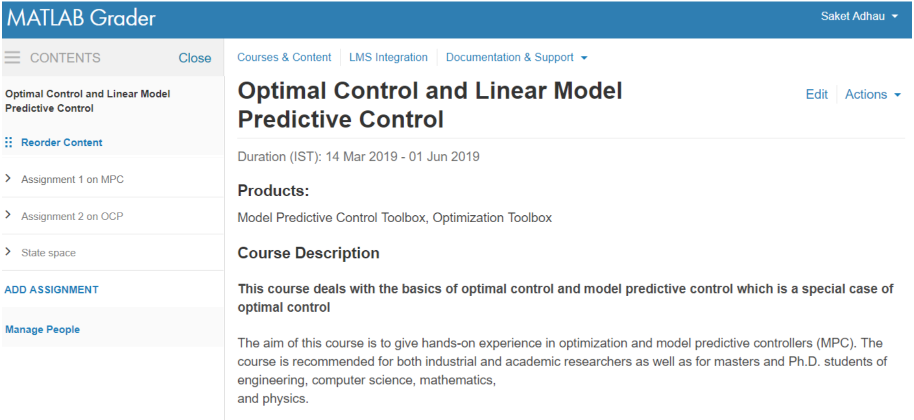
```
x_new =
```

$$\begin{pmatrix} -1 \\ \frac{3}{2} \end{pmatrix}$$

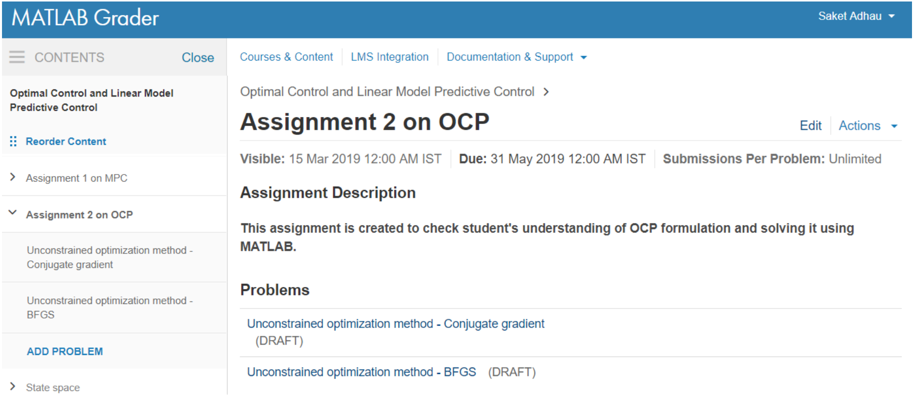
```
fval =
```

$$-\frac{5}{4}$$

Assessing the students using MATLAB Grader



Assessing the students using MATLAB Grader

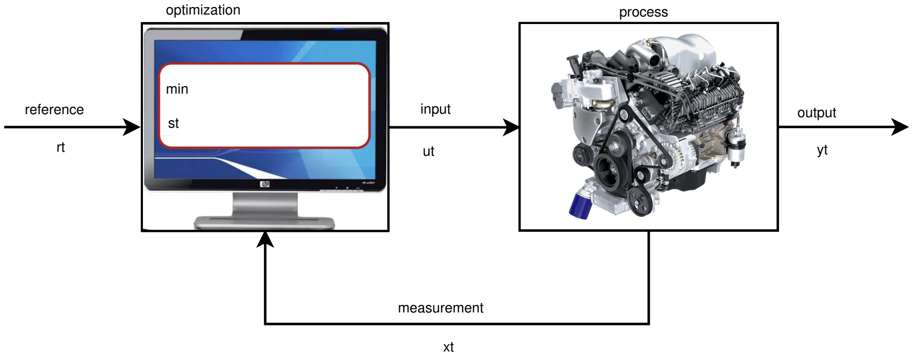


Assessing the students using MATLAB Grader: Report

Case Study:

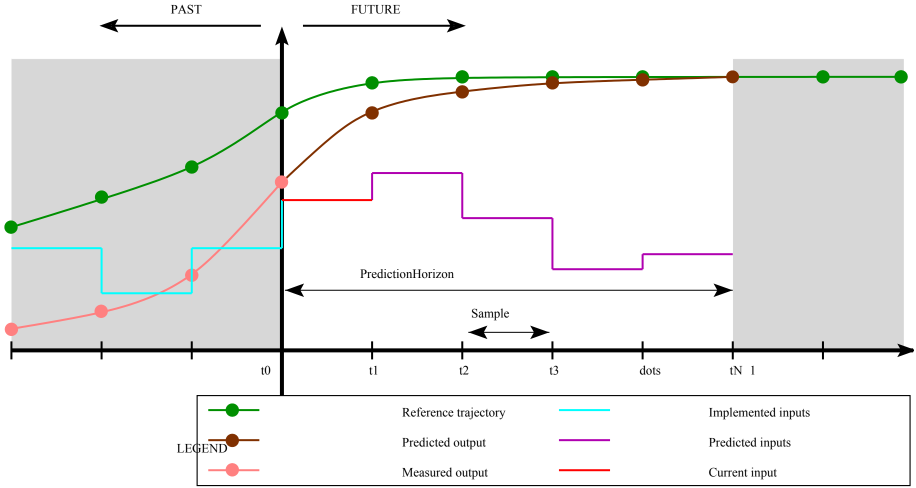
- We have a class of **20** students opting for **Process Modeling and Optimization**.
- As an instructor, it is one challenging task to teach students MATLAB based coding interactively and assessing them individually.
- MATLAB has made this task easier than ever, by introducing MATLAB Live script and MATLAB Grader.

Optimal Control Problem leads to Model Predictive Control (MPC)



Use a dynamical model of the process to predict its future evolution and choose the "best" control action

Receding Horizon Implementation



Formulating the state space model for MPC

Create a state space model of the plant and set some of the optional model properties.

The State Space model for DC Motor is described as,

$$\begin{bmatrix} \dot{i_a(t)} \\ \dot{\omega(t)} \\ \dot{\theta(t)} \end{bmatrix} = \begin{bmatrix} \frac{-R_a}{L_a} & \frac{-K_b}{L_a}\omega(t) & 0 \\ \frac{-K_t}{J} i_a & \frac{-f}{J}\omega(t) & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a(t) \\ \omega(t) \\ \theta(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_a(t) \\ \omega(t) \\ \theta(t) \end{bmatrix}$$

Define the state space model in matlab script.

```
A =  [-Ra/La,-km/La;km/J,-fm/J]
```

```
A = 2×2
10^3 ×
     -7.2414   -0.0354
      1.0524   -0.0000
```

```
B =  [1/La;0]
```

```
B = 2×1
    862.0690
           0
```

Create Controller

Create a model predictive controller with a sample time, of 0.0001 second, and with all other properties at their default values.

```
Ts = 0.001;

MPCobj = mpc(motor,Ts);
```

```
-->Assuming unspecified output signals are measured outputs.
-->The "PredictionHorizon" property of "mpc" object is empty. Trying PredictionHorizon = 10.
-->The "ControlHorizon" property of the "mpc" object is empty. Assuming 2.
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming default 0.00000.
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming default 0.10000.
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1.00000.
   for output(s) y1 and zero weight for output(s) y2
```

Display the controller properties in the Command Window.

```
display(MPCobj)
```

```
MPC object (created on 22-Mar-2019 16:22:13):
---------------------------------------------
Sampling time:      0.001 (seconds)
Prediction Horizon: 10
Control Horizon:    2

Plant Model:
                                      --------------
      1  manipulated variable(s)   -->|  2 states  |
                                      |            |-->  2 measured output(s)
      0  measured disturbance(s)   -->|  1 inputs  |
                                      |            |-->  0 unmeasured output(s)
      0  unmeasured disturbance(s) -->|  2 outputs |
                                      --------------
Disturbance and Noise Models:
        Output disturbance model: default (type "getoutdist(MPCobj)" for details)
         Measurement noise model: default (unity gain after scaling)
```

Teaching students MPC using Live Editor

View and Modify Controller Properties

Display a list of the controller properties and their current values.

```
get(MPCobj)
```

```
                           Ts: 0.001
       PredictionHorizon (P): 10
          ControlHorizon (C): 2
                        Model: [1x1 struct]
    ManipulatedVariables (MV): [1x1 struct]
         OutputVariables (OV): [1x2 struct]
    DisturbanceVariables (DV): []
                  Weights (W): [1x1 struct]
                    Optimizer: [1x1 struct]
                        Notes: {}
                     UserData: []
                      History: 22-Mar-2020 16:22:13
```

The controller is set with default properties, we will modify them according to our purpose.

```
MPCobj.PredictionHorizon = 10;
MPCobj.ControlHorizon = 5;
```

By default, the controller has no constraints on manipulated variables and output variables. Set co

```
MPCobj.MV.Min = 0;
MPCobj.MV.Max = 18;
```

Modify the MPC Properties.

Weights on inputs and outputs state variables.

```
MPCobj.W.ManipulatedVariablesRate = 0.1;
MPCobj.W.OutputVariables = [0 1];
```

Teaching students MPC using Live Editor

Simulating the controller

Time for running the simulation, (10000 control intervals)

```
T = 10000;
```

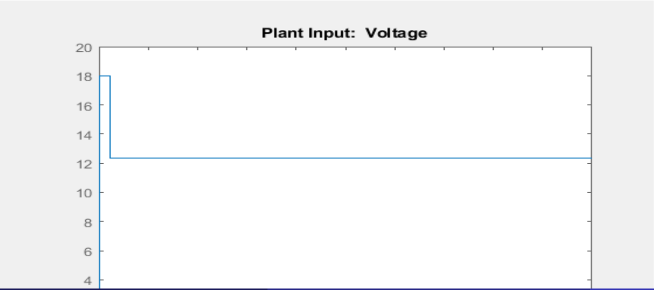
Specify setpoints of 0 and 300 for the Current and the Speed respectively. The setpoint for the Current is ignored because

```
r = [0 0 ; 0 300]
```

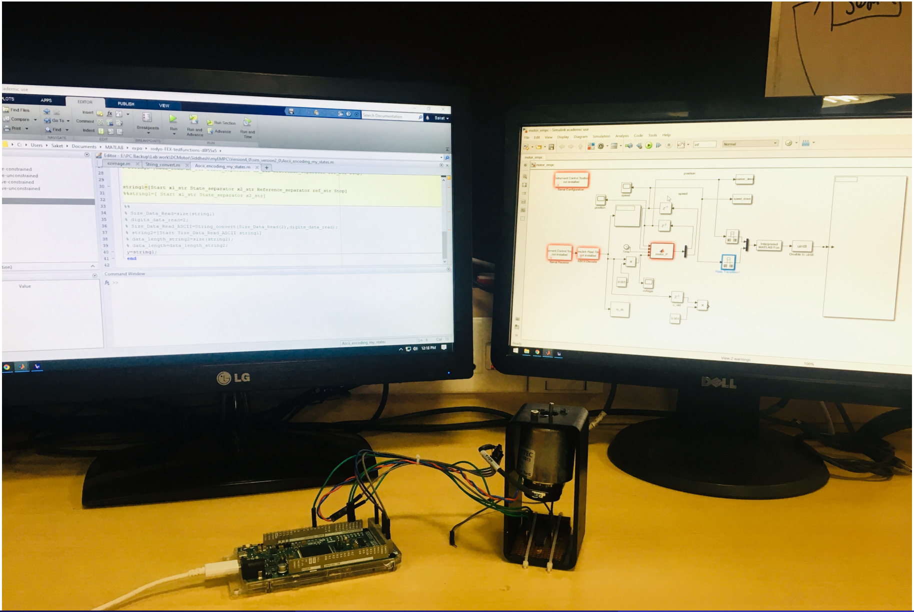
```
r = 2×2
     0     0
     0   300
```

```
sim(MPCobj,T,r)
```

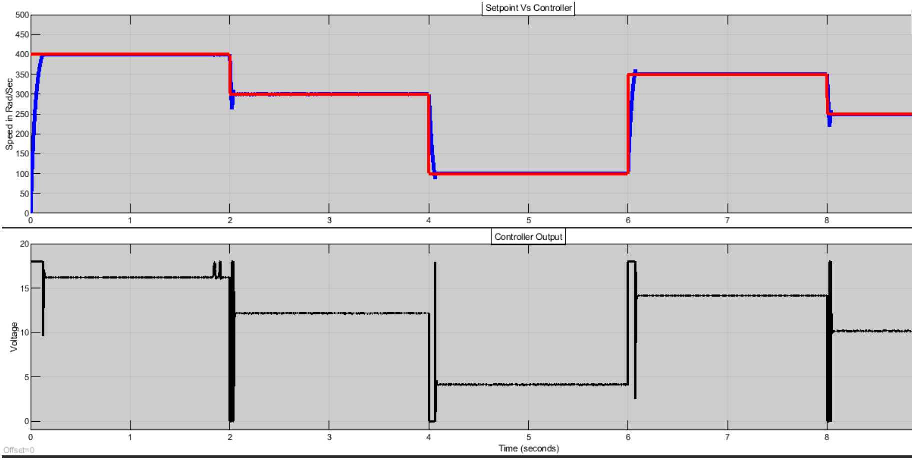
```
-->Converting model to discrete time.
-->Assuming output disturbance added to measured output channel #2 is integrated white noise.
-->Assuming output disturbance added to measured output channel #1 is integrated white noise.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on each measured output channel.
```



Live Demo using Simulink and Arduino



Live Demo using Simulink and Arduino



The END