Numerical Simulation of Tsunami on GPU

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TSUNAMI

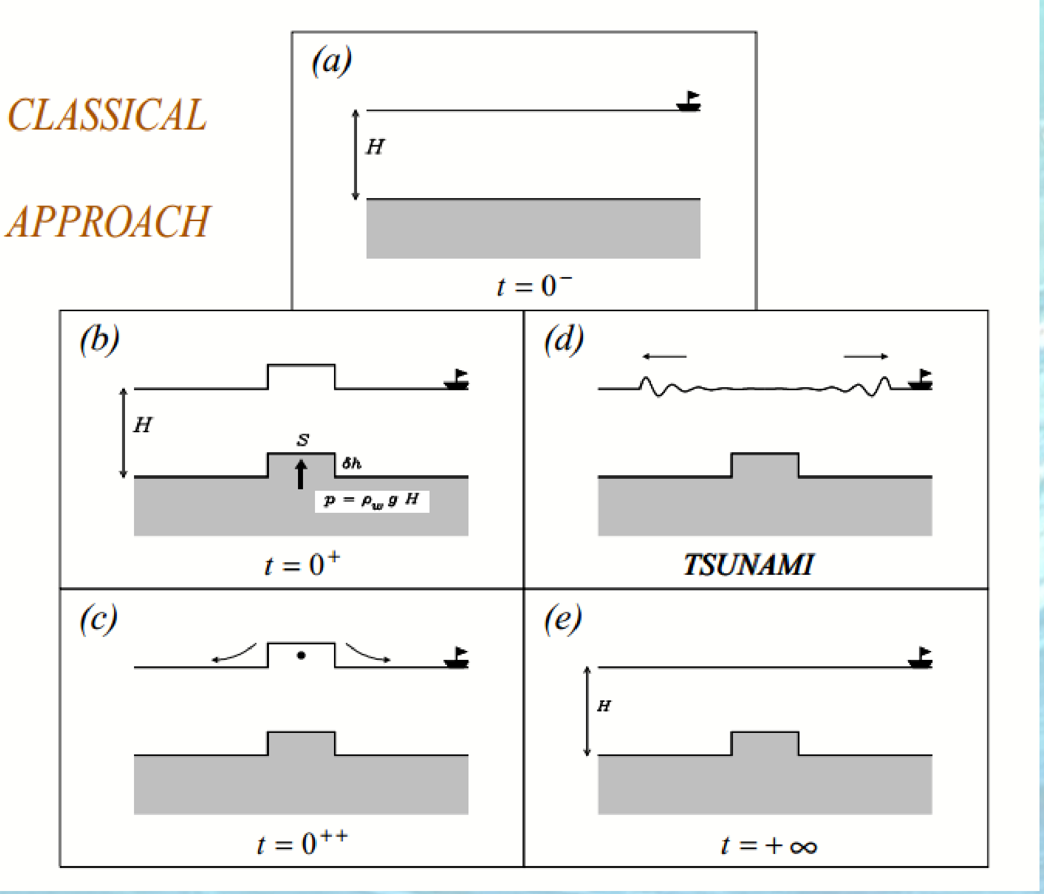
Propagation of disturbance on the ocean free surface under gravitational force from source to coast.

TSUNAMIS GENERATED BY

- ❖**Earthquakes**
- ❖**Landslides**
- ❖**Volcanic Explosions**
- ❑**Meteo – Tsunamis**

Tsunami Simulation Steps

1. Generation
2. Propagation
3. Run Up/ Inundation



Classical Approach: Source – Prof. Emile A. Okal, Northwestern University, 2017.

Governing Equations

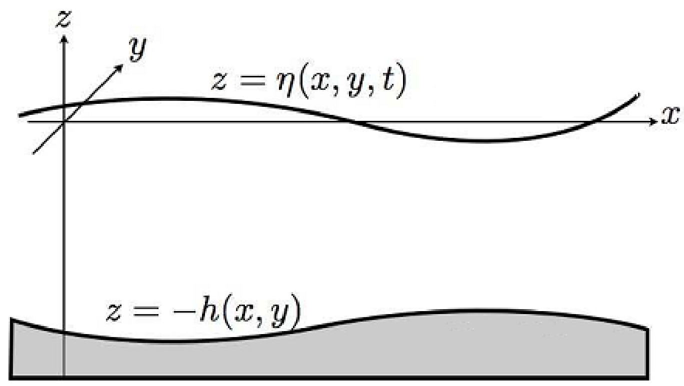
λ >> h

Non linear Shallow water equations

$$\frac{\partial \eta}{\partial t}+\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}=0$$

$$\frac{\partial M}{\partial t}+\frac{\partial}{\partial x}\left(\frac{M^2}{D}\right)+\frac{\partial}{\partial y}\left(\frac{MN}{D}\right)+gD\frac{\partial \eta}{\partial x}+\frac{gn^2}{D^{7/3}}M\sqrt{M^2+N^2}=0$$

$$\frac{\partial N}{\partial t}+\frac{\partial}{\partial x}\left(\frac{MN}{D}\right)+\frac{\partial}{\partial y}\left(\frac{N^2}{D}\right)+gD\frac{\partial \eta}{\partial y}+\frac{gn^2}{D^{7/3}}N\sqrt{M^2+N^2}=0$$



Linear Shallow water equations

$$\frac{\partial \eta}{\partial t}+\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}=0$$

$$\frac{\partial M}{\partial t}+gD\frac{\partial \eta}{\partial x}=0$$

$$\frac{\partial N}{\partial t}+gD\frac{\partial \eta}{\partial y}=0$$

$\eta \rightarrow$ Wave height

$M, N \rightarrow$ discharge fluxes in x and y directions

$D \rightarrow$ Total water depth

$n \rightarrow$ Mannings roughness coefficient

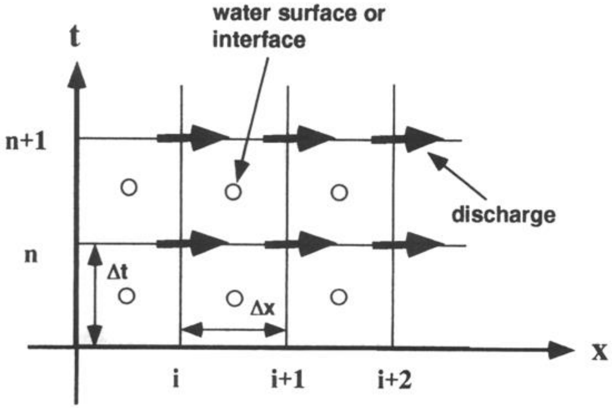
$M = u(h+D) = uD; N = v(h+D) = vD$

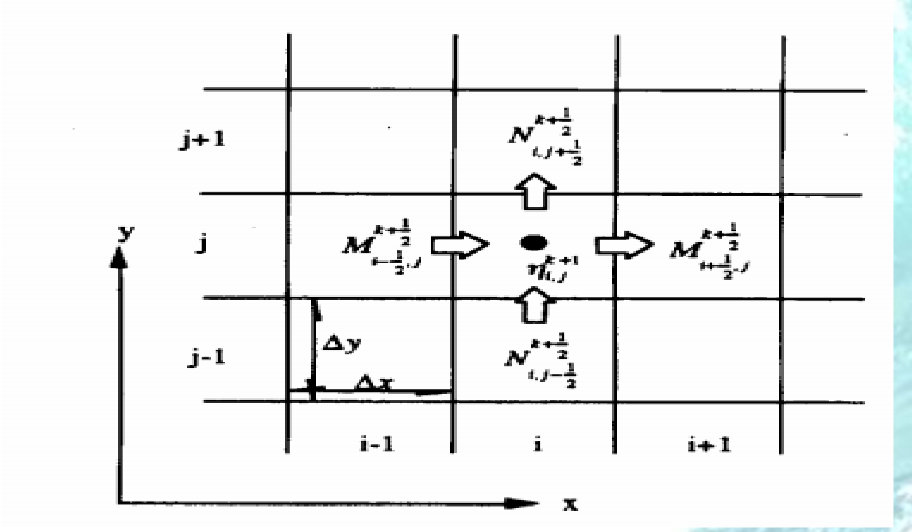
$$n=\sqrt{\frac{fD^{1/3}}{2g}}$$

$u, v \rightarrow$ velocity in x and y directions

$f \rightarrow$ friction coefficient

Numerical Methodology





1. Initial boundary value problem
2. Equations are solved by employing finite difference method (FDM)
3. The staggered leap-frog scheme (Shuto, Goto, Imamura, (1990)) is used to solve the governing equations.
4. At every time step, wave is propagated by calculating the water surface elevations and water velocities throughout the domain.
5. For stability the time step and grid size should be selected properly and they should obey CFL condition

$$\frac{\nabla x}{\nabla t} = \sqrt{2gh_{\max}}$$

Finite Difference Scheme

$$\frac{\partial \eta}{\partial t} = \frac{1}{\Delta t}\left[\eta_{i,j}^{k+1} - \eta_{i,j}^{k}\right]; \frac{\partial M}{\partial x} = \frac{1}{\Delta x}\left[M_{i+\frac{1}{2},j}^{k+\frac{1}{2}} - M_{i-\frac{1}{2},j}^{k+\frac{1}{2}}\right]; \frac{\partial N}{\partial y} = \frac{1}{\Delta y}\left[N_{i,j+\frac{1}{2}}^{k+\frac{1}{2}} - N_{i,j-\frac{1}{2}}^{k+\frac{1}{2}}\right]$$

$$\eta_{i,j}^{k+1} = \eta_{i,j}^{k} - \frac{\Delta t}{\Delta x}\left[M_{i+\frac{1}{2},j}^{k+\frac{1}{2}} - M_{i-\frac{1}{2},j}^{k+\frac{1}{2}}\right] - \frac{\Delta t}{\Delta y}\left[N_{i,j+\frac{1}{2}}^{k+\frac{1}{2}} - N_{i,j-\frac{1}{2}}^{k+\frac{1}{2}}\right]$$

→ **Wave height**

$$M_{i+\frac{1}{2},j}^{k+\frac{1}{2}} = M_{i+\frac{1}{2},j}^{k-\frac{1}{2}} - gD_{i+\frac{1}{2},j}^{k}\frac{\Delta t}{\Delta x}\left[\eta_{i+1,j}^{k} - \eta_{i,j}^{k}\right]$$

$$D_{i+\frac{1}{2},j}^{k} = h_{i+\frac{1}{2},j} - \eta_{i+\frac{1}{2},j}^{k} = h_{i+\frac{1}{2},j} + \frac{1}{2}\left[\eta_{i+1,j}^{k} + \eta_{i,j}^{k}\right]$$

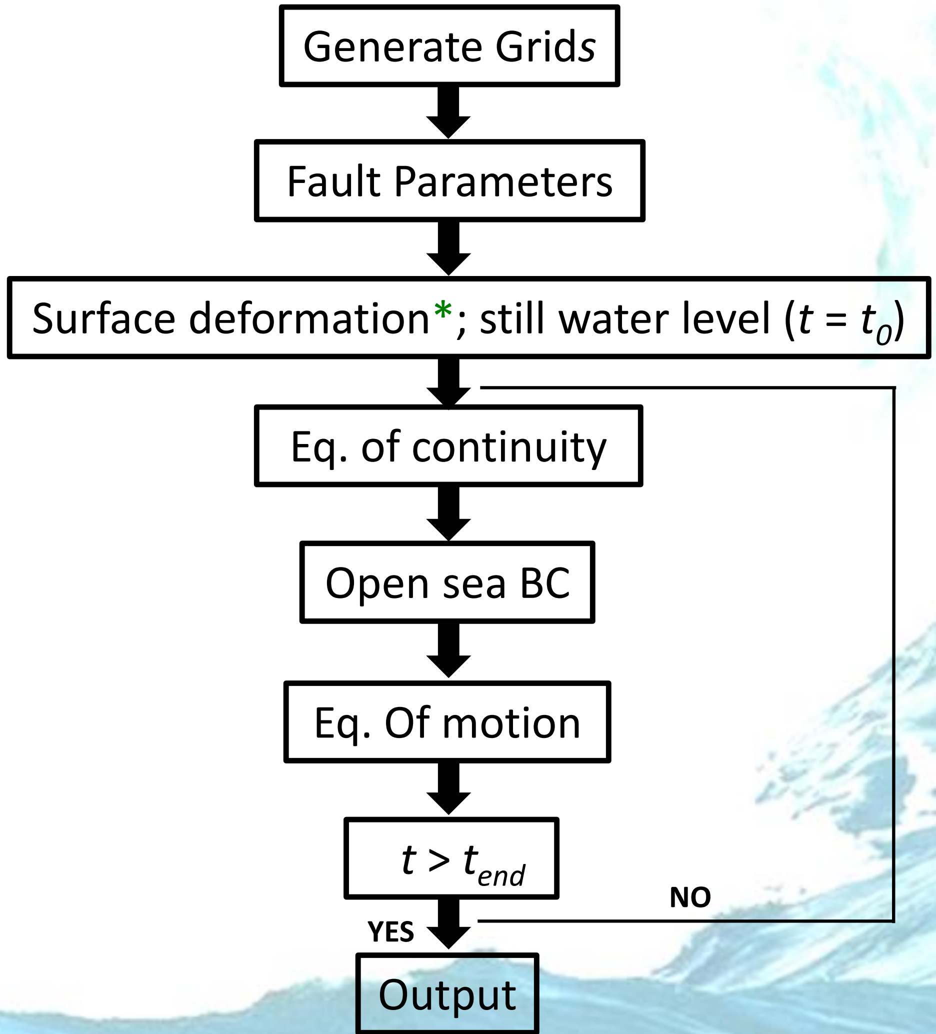
→ **Discharge flux along x**

$$N_{i,j+\frac{1}{2}}^{k+\frac{1}{2}} = N_{i,j+\frac{1}{2}}^{k-\frac{1}{2}} - gD_{i,j-\frac{1}{2}}^{k}\frac{\Delta t}{\Delta y}\left[\eta_{i,j+1}^{k} + \eta_{i,j}^{k}\right]$$

$$D_{i,j+\frac{1}{2}}^{k} = h_{i,j+\frac{1}{2}} + \eta_{i,j+\frac{1}{2}}^{k} = h_{i,j+\frac{1}{2}} + \frac{1}{2}\left[\eta_{i,j+1}^{k} + \eta_{i,j}^{k}\right]$$

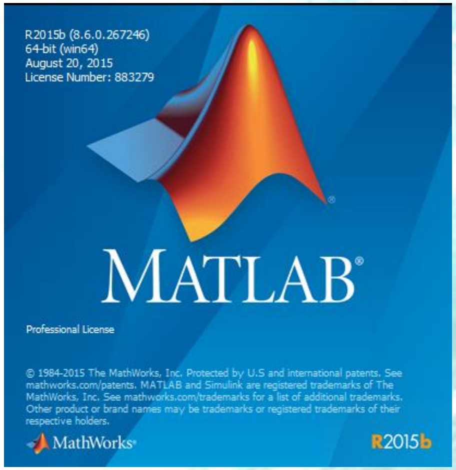
→ **Discharge flux along y**

Algorithm

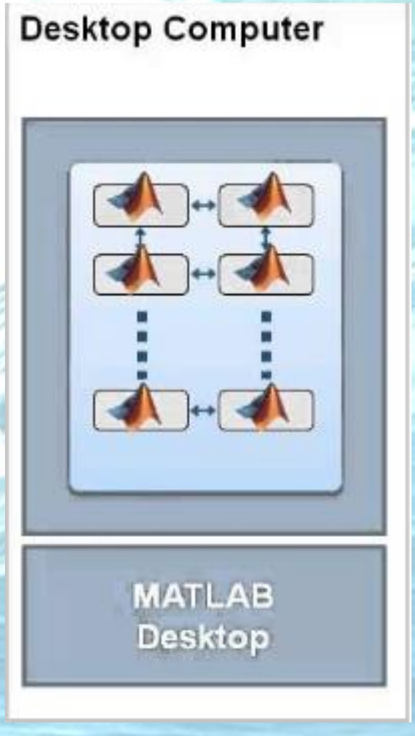

Generate Grids
Fault Parameters
Surface deformation*; still water level ($t = t_0$)
Eq. of continuity
Open sea BC
Eq. Of motion
$t > t_{end}$
NO
YES
Output

Tools Used

1. MATLAB 2015b
2. Parallel Computing toolbox
3. NETCDF tool box



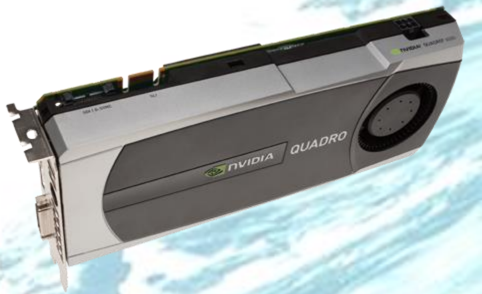
The Finite Difference Scheme is coded in MATLAB and run on GPU



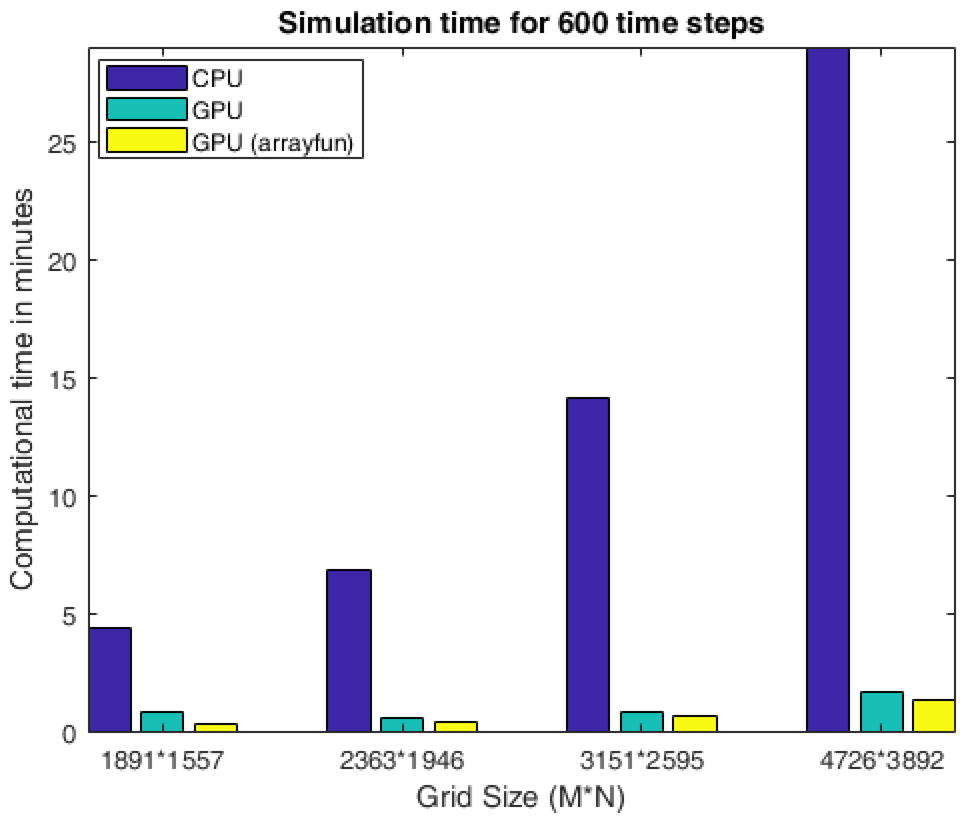
GPU Device

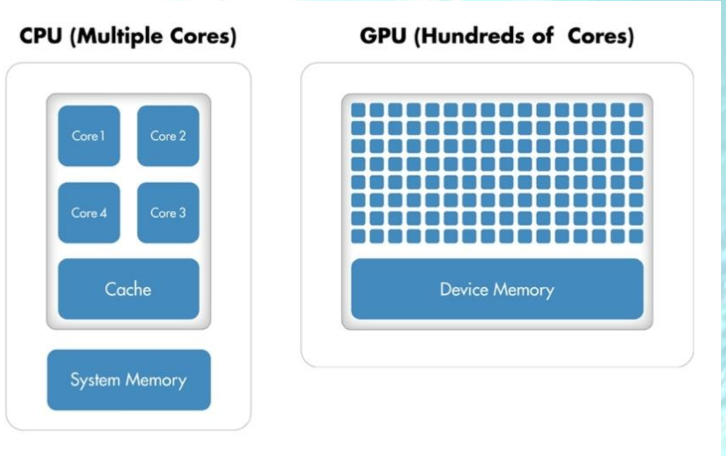
```
CUDADevice with properties:

                      Name: 'Quadro K6000'
                     Index: 1
         ComputeCapability: '3.5'
            SupportsDouble: 1
             DriverVersion: 8
            ToolkitVersion: 8
        MaxThreadsPerBlock: 1024
          MaxShmemPerBlock: 49152
        MaxThreadBlockSize: [1024 1024 64]
               MaxGridSize: [2.1475e+09 65535 65535]
                 SIMDWidth: 32
               TotalMemory: 1.2885e+10
           AvailableMemory: 1.2622e+10
       MultiprocessorCount: 15
              ClockRateKHz: 901500
               ComputeMode: 'Default'
      GPUOverlapsTransfers: 1
    KernelExecutionTimeout: 1
          CanMapHostMemory: 1
           DeviceSupported: 1
            DeviceSelected: 1
```



Source: NVIDIA Home Page



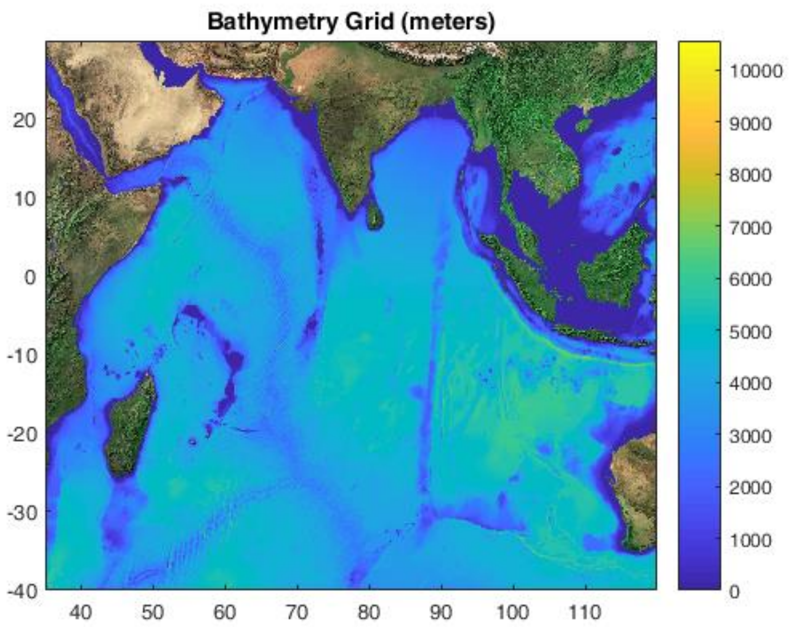


Source: Mathworks

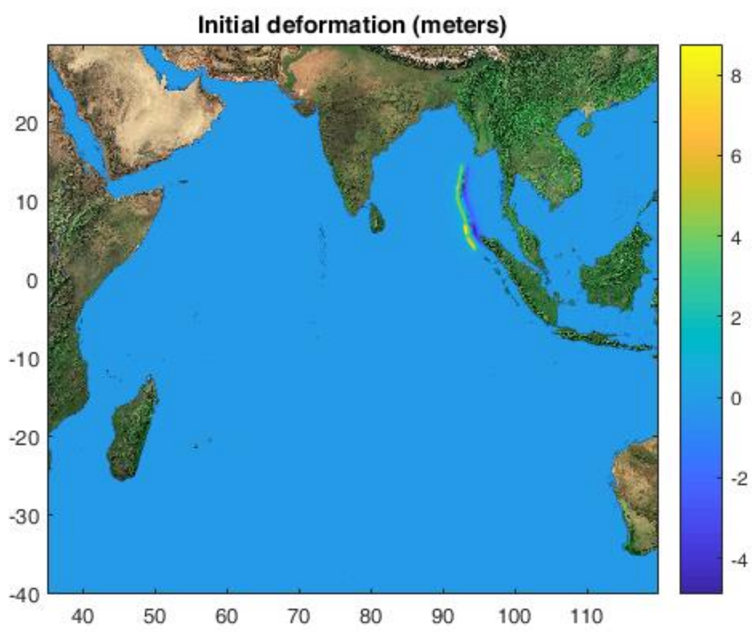
Bar plot of benchmark results showing the time required to complete 600 time steps for different grid sizes on CPU and GPU

Results

Generation



Gebco 30arc second Bathymetry (2 Kms Resolution)
Grid size - 4726*3892

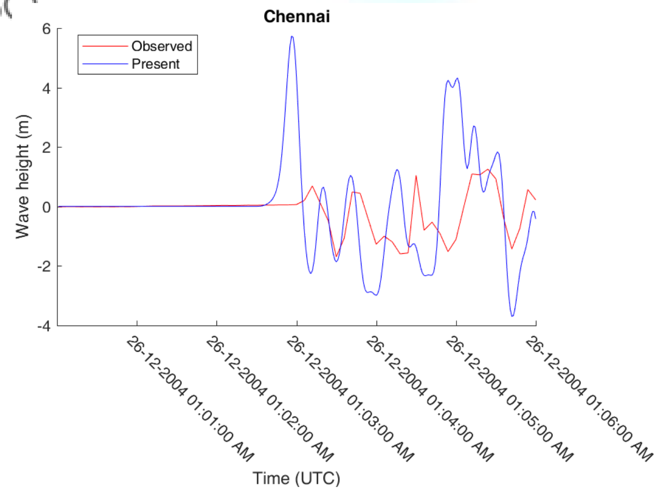


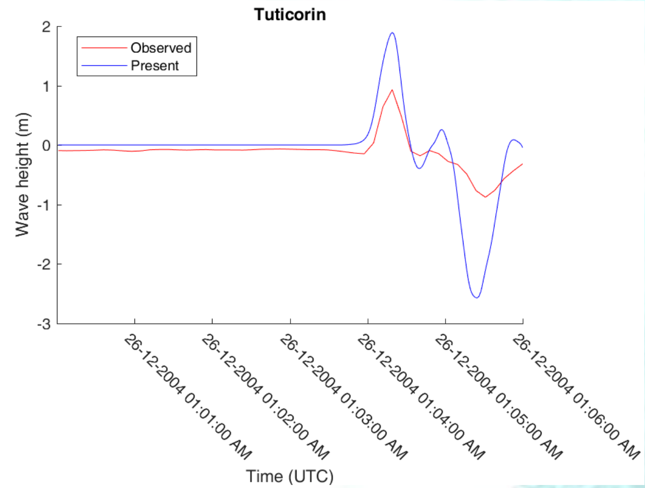
Initial deformation Dec 26, 2004 Tsunami

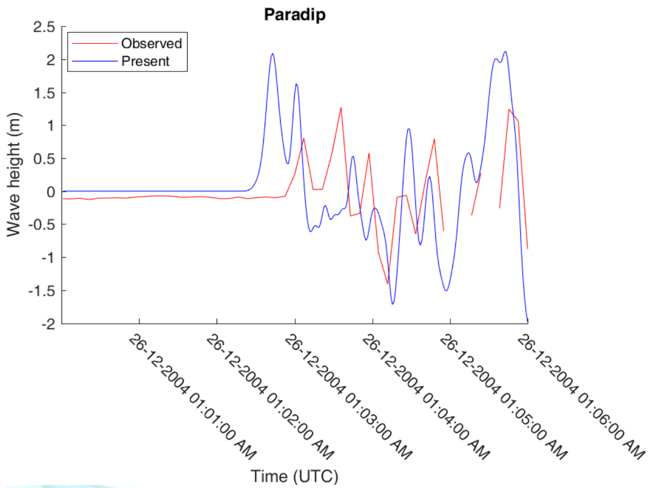
Mansinha, L. and Smylie, D.E, 1971. "The displacement fields of inclined faults" Bulletin of the Seismological Society of America, Vol. 61, 1433-1440.

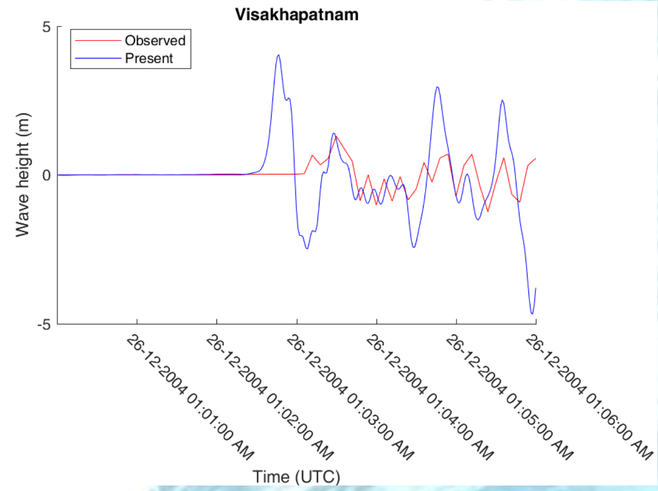
Grilli, S. T., Ioualalen, M., Asavanant, J., Shi, F., Kirby, J., and Watts, P.: Source constraints and model simulation of the December 26, 2004 Indian Ocean tsunami, J. Waterway Port Coastal and Ocean Engineering, 133(6), 414–428, 2007.

Propagation….



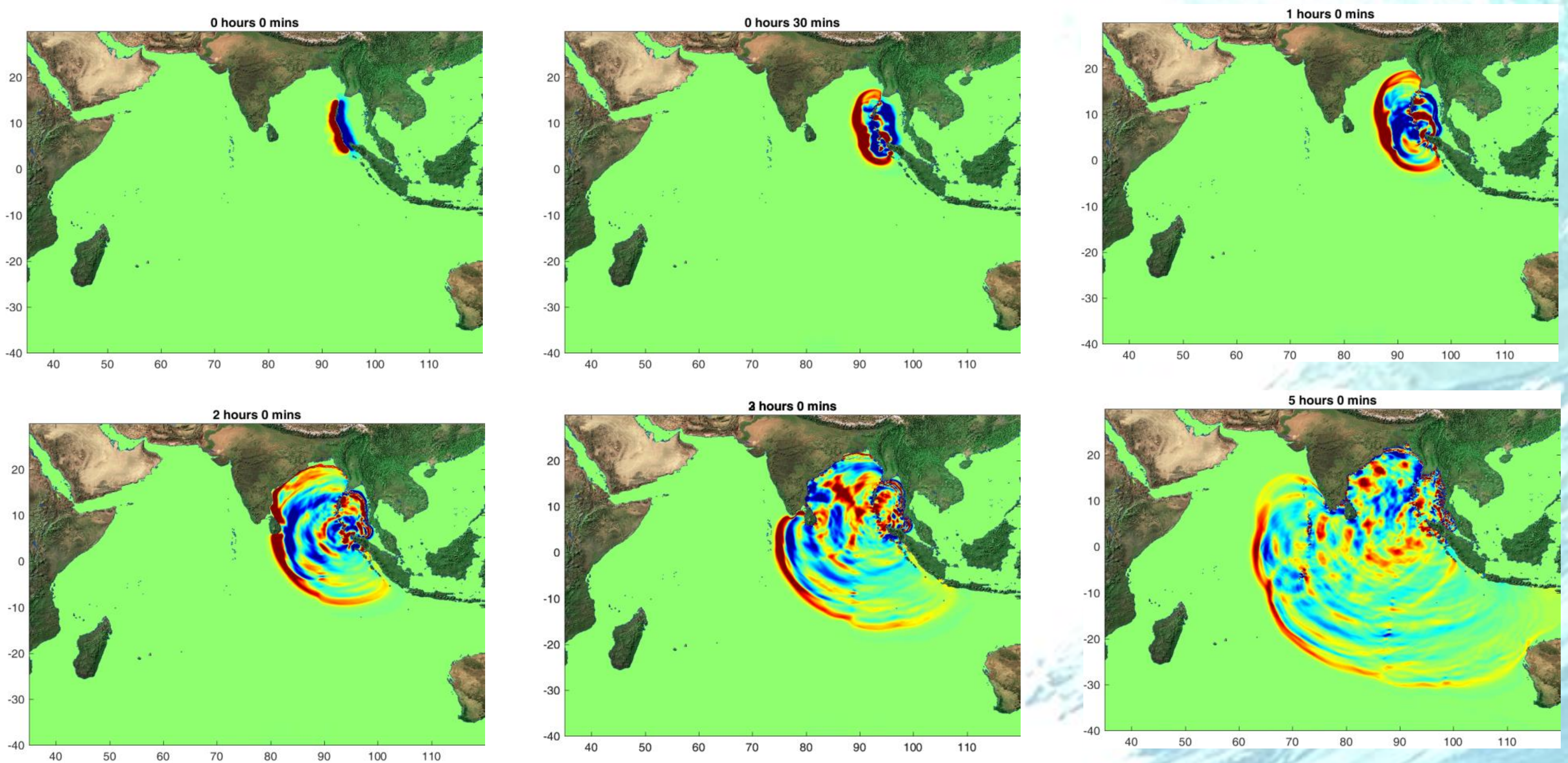






Comparison of present result with real time observations

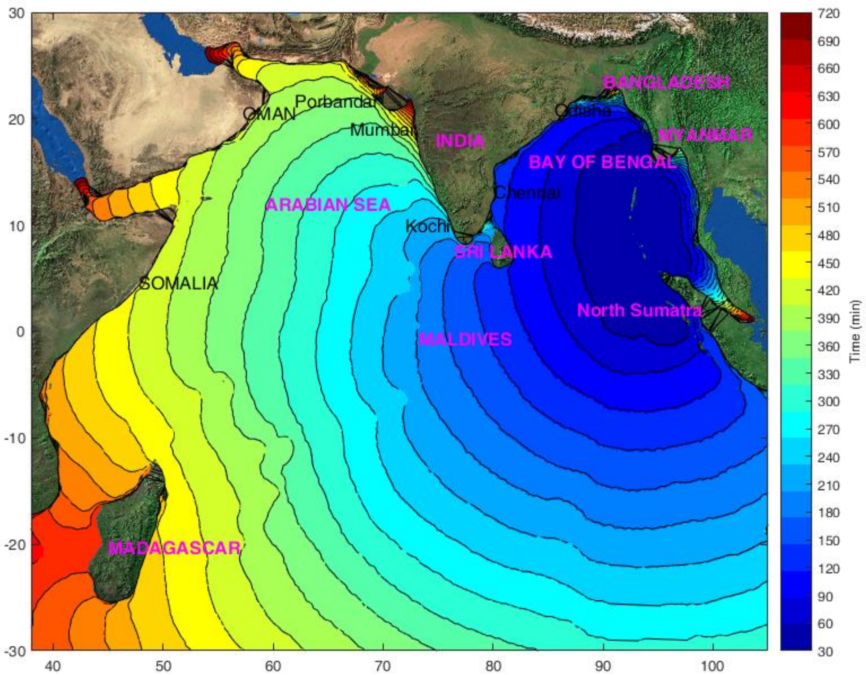
Propagation



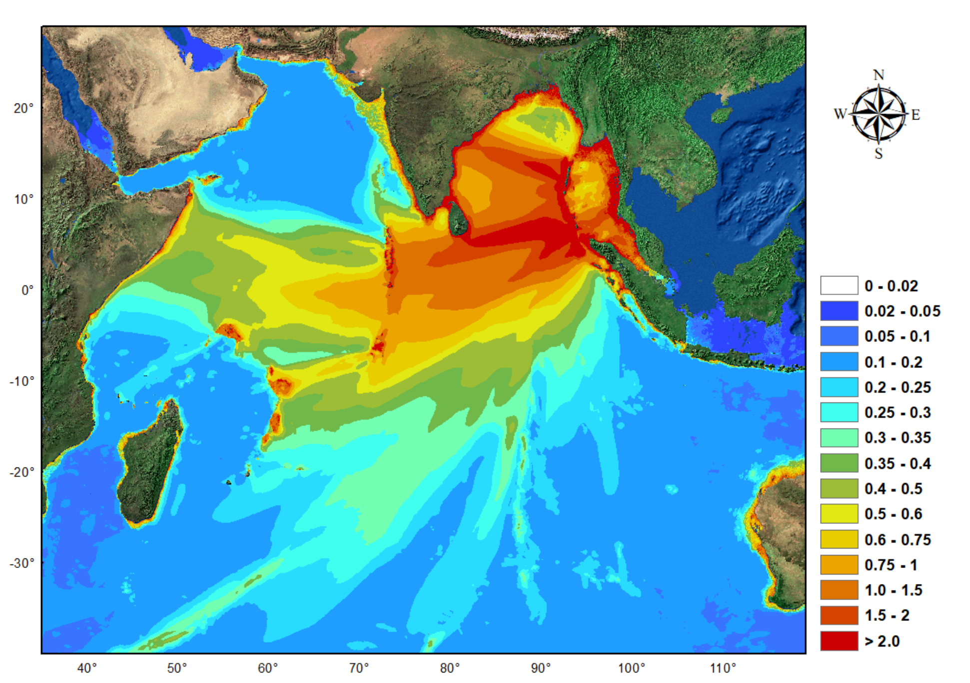
Wave propagation at different time stamps

Animation of the above propagation can be viewed on: https://youtu.be/RIFYSbeZji8

Propagation....



Travel Times
(One Hour Interval)



Directivity Plot
(Wave height in meters)

Conclusion and Future Work

1. Open ocean propagation of tsunami is simulated employing Finite Difference Method
2. A good speed is achieved by using MATLAB on GPU
3. GPU coding can be implemented with ease using GPU for MATLAB
4. Pre and Post processing of results is comfortable in the MATLAB environment
5. The results of present code are in good agreement with observed data
6. The present code can be used for academic purpose and for data base generation

1. Run up/ inundation should be included
2. Further speed has to be achieved to use the present code for operational purpose
3. Simulate Tsunami for Global grid
4. Employ Wavelet and FFT based techniques available in MATLAB to analyse Tsunami data

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